Reliability Centred Sparing – Part 2: Multi-Echelon Sparing

In SERT 1-4 we described how to apply reliability theory to help predict the sparing level required for a high cost, repairable system. Using the equipment MTBF and some statistical theory, we were able to model the Risk of Shortage (ROS) and Expected Back Orders (EBO) that would emerge for different sparing levels. By selecting an appropriate cost function we were able calculate the minimum level of spares required to achieve an optimal outcome.

The support model used in SERT 1-4 assumed a single depot for the system which held the spares and also repaired faulty equipment. However, the support chain is often more complex than this and so this SERT Sheet will consider a multi-echelon support model. In this model a system is supported at one or more 'forward bases' which hold a local stock of spares. The forward bases have no repair facility. A central depot maintains the majority of spares and also performs repair on the failed equipment as it is received from a forward base. The general sequence is: a part fails, it is replaced from local stock, the faulty part is shipped to the depot, the depot ships a replacement spare to the base, the depot puts the faulty part into the repair facility.

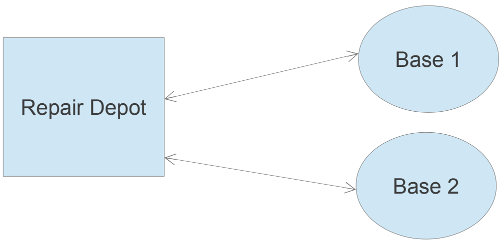

Figure 1. 2-Echelon Repair

This model is illustrated in Figure 1. It is assumed that the time to ship an item from the base to the depot (T_{bd}) is known, and also from the depot to the base (T_{db}). The repair time is a random variable with expected value $T_{rep.}$ The problem now is to determine how many spares to hold at the depot and how many to hold in each forward base in order to achieve an optimum sparing solution. Once again, the definition of 'optimum' will depend on how the sparing penalty is calculated.

At the Depot

Faulty items arrive to the depot according to a Poisson process with intensity $\lambda_0 = \lambda_1 + \lambda_2$, where λ_i is the intensity at base *i*. If X_0 represents the number of items in, or on their way to, the repair facility then the expected value of X_0 is:

$$E[X_0] = \lambda_0 T_0, \text{ where } T_0 = T_{bd} + T_{rep}.$$

With this information the expected number of back orders (EBO) at the depot can be calculated using the approach described in SERT 1-4.

At the Base

We want an expression for the average number of faulty items at a base that are in the repair pipeline at any given time. These items come from two sources: the number of faults that have occurred at the base within the last T_{db} time units, and so haven't been replaced yet, and the items that failed more than T_{db} time units ago but were back orders at the depot T_{db} time units ago. If X_j is the number of items in the repair pipeline from base j then:

$$E[X_j] = \lambda_j \left(T_{db} + \frac{EBO(s_0)}{\lambda_0} \right)$$

where $EBO(s_0)$ is the total number of expected back orders at the depot.

From Palm's Theorem X_j is a Poisson random variable, and hence

$$P(X_j = k) = \frac{(X_j T_j)^k}{k!} e^{-\lambda_j T_j}$$

where $T_j = T_{db} + \dfrac{EBO(s_0)}{\lambda_0}$.

This means it is possible to calculate the average number of back orders at base j using the recursive equations developed in SERT 1-4.

Algorithm

To calculate the optimum spares holding we must select a suitable cost function. Our aim may be to minimise cost, or minimise the risk of shortage, or some other measure. For example, let's assume that we have two bases and one supporting depot and we want to stock the system with the minimum stock to ensure that the risk of shortage at the bases is less than 0.5%. For simplicity we will assume that the repair parameters are the same at each base and each base supports half the fleet. The parameters are:

a. Fleet size per base = 400
b. MTBF = 20,000 hours
c. $T_{db} = T_{bd}$ = 7 days
d. Average repair time at depot = 30 days
e. S_0 = spares held at depot
f. S_b = spares held at each base

Step 1. Set $s_0 = 0$ and calculate s_b such that ROS < 0.5%. (S_0=0, s_b=33, Total=66, ROS=0.39%)

Step 2: Increase the number of spares held at the depot by one, then re-calculate s_b for ROS < 0.5%. (s_0=1, s_b=32, Total=65, ROS=0.46%)

Step 3: Repeat Step 2 until the minimum total number of items is achieved. (s_0=35, s_b=8, Total=51, ROS=0.43%)

Therefore, the minimum number of spares required to achieve a risk of shortage at the bases of less than 0.5% is 51 items. They should be distributed with 35 at the depot and 8 at each of the two bases. This provides a saving of 15 items when compared with just stocking the bases, which can be a considerable saving when each spare item is expensive.

About the Author:

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