Reliability Centred Sparing – Part 1: A Single Depot

The spares inventory can be a significant cost driver for any project, but particularly when high value components are required with relatively low failure rates. If too many spares are held then there is an excessive investment in inventory. If too few spares are held then the operational system may experience excessive operational downtime waiting for new spares to be purchased and delivered.

Measuring the Cost

For each project the spares inventory should reflect the notion of *value* for the project. In other words, the cost function that is used to determine the optimum spares holdings can vary from project to project, depending on how a lack of spare parts creates a penalty for the customer or the producer.

For example, the sparing strategy may aim to minimise the operational downtime of the deployed system. Alternately, the sparing strategy may aim to ensure that certain stock holdings at certain locations do not drop below minimum KPI values. Or the aim might be to minimise costs through trading the cost of permanent warehouse space against the cost of fast shipping when a failure occurs.

It is sensible to have a well defined, systematic approach for calculating the required spares holding and their location. This SERT Sheet describes a mathematical model for calculating spare holdings based on the reliability data of the system components.

Support Model

Single Echelon Support

The size of the spare holdings will depend on the support model that will be implemented in the field. For example, a system might be supported from a single depot with a repair facility, with all failed items returning to this facility. This support model could be appropriate when the deployed system is only operating in one location.

A reliability-centred sparing model will calculate the number of spares that need to be held at the depot by using the system failure data and the mean time to repair the item in the workshop. Under this model if an item fails then it is immediately swapped for a spare so that the system can remain operational. The faulty item enters the workshop for a period of time and then emerges to once again join the spares inventory.

Multi-Echelon Support

A multi-echelon support arrangement could consist of a central depot with a repair facility and one or more forward bases. Each base holds spare parts, but has no repair workshop. Under this model a reliability-centred approach would calculate the number of spares to hold at the depot and the number to be held at each forward base. The model would consider the number of systems supported from each base, turn-around time from the base to the depot, as well as the repair time once the item is at the depot.

The number of spares required is dependent on whether a single or multi-echelon support model is used. This SERT Sheet will introduce the basic ideas using a single depot model and SERT 1-5 will develop reliability-centred sparing for a multi-echelon model.

Key Parameters

The first step is to calculate some key parameters that we will use in the model.

Expected Number of Failed Items in the Repair Pipeline

For the purpose of this sparing model the supported system is assumed to have an exponential failure distribution. The exponential distribution is commonly assumed for a wide variety of complex systems. Given an exponential failure performance, the probability of a particular number of failed items, *k*, being in the repair pipeline is calculated by the Poisson distribution:

$$Pr(X=k)=\frac{\lambda^k e^{-\lambda}}{k!}$$

where λ is the mean number of failed items expected in the pipeline. Clearly, being able to predict the likelihood of this number is important in estimating the number of spares required. Therefore, the first parameter we need is an estimate of the expected (or average) number of failed items in the repair pipeline at any given time.

The number of items in the repair pipeline is a function of the failure rate of the system and also the repair rate within the workshop. To calculate the failure rate a typical system will have an operational profile that tells us the proportion of utilised hours. For example, a device might be used eighteen hours per day, which provides a utilisation of 0.75. Or an aircraft might fly 500 hours per month, which is a utilisation of 0.69. Each system will also have a defined Mean Time Between Failure (MTBF). If we have a fleet of systems of size *M* with average utilisation *U* then the average failure rate across the fleet at any given moment is calculated as:

$$Fleet\ Fail\ Rate = \frac{M * U}{MTBF}$$

The repair rate, μ, must be defined for the workshop, or is represented by the turn around time if the failed items are sent to an OEM for repair. If we assume that the workshop always has sufficient capacity then the mean number of items in the repair pipeline can be calculated using Palms Theorem as

$$\lambda = (Fleet\ Fail\ Rate) * \mu$$

With this calculation we can now work out the probability of *k* failures being in the repair pipeline at any given moment.

Risk of Shortage (ROS)

The ROS is the probability that the spares holding will be less than the minimum permitted holding. In simple cases this simply means the risk of needing *s+1* or greater spares when you are only holding *s* spares. However, if you must keep minimum holdings of *p* spares then the ROS is the probability of needing *s-p+1* spares or greater.

Clearly, the ROS is a function of *s*, the number of spares on hand. Using the Poisson distribution,

$$ROS(s) = 1 - \sum_{i=0}^{s} \frac{\lambda^i e^{-\lambda}}{i!}$$

Expected Back Orders (EBO)

In the single depot support model, when a failure occurs we swap the failed item from the system for one from the spares holding at the depot, in order to put the system immediately back into service. The failed item then goes into the depot workshop and progresses through the repair pipeline. After some time, there is a risk that we might experience a failure and not have any spares left to swap into the system. At this point we have a Back Order, and we must wait for the next repaired item to emerge from the workshop. Since a Back Order is equivalent to having a dormant system, or grounded aircraft, the Expected Back Orders (EBO) is the average number of systems waiting for parts at any given time. Clearly, EBO is a function of *s*, where *s* is the number of spares held at the depot. The more spares we hold, the lower the average number of Back Orders that will occur.

It can be shown that calculating the EBO for a given value of *s* is a recursive function that uses the ROS.

EBO(s+1) = EBO(s) - ROS(s) for s=0,1,2,...

Applying a Cost Function

Let's assume the system is a fleet of 500 radios with a MTBF of 50,000 hours and a utilisation of 0.75. Let's also assume that the radios are returned to an OEM for repair, with a turn around time of 50 days. Hence,

$$\lambda = \frac{M * U}{MTBF} \mu = 9\ radios$$

We are now in a position to calculate the optimum number of spares to hold at the depot. However, to do that we need to define what *optimum* means. Let's consider two different cost functions.

Option 1: Holdings KPI

For this example we assume that the contract has a KPI that will be penalised if the number of spares falls to zero more than 5% of the time. In other words, we want the ROS to be less than 5% with a minimum holding of one item.

Using a spreadsheet we can compute *ROS(s)* for different values of *s* and select the smallest *s* such that ROS < *0.05.* When we do this we find that the depot needs to hold 15 spare radios.

Option 2: Minimise Operational Downtime

If the radio is a critical element in a vehicle then another optimisation might be to minimise the operational downtime of the supported vehicle. For example, consider this cost function, which measures the total cost of parts for the repair cycle.

*g(s) = c*s + q*EBO(s)*

where *c* is the cost of the spare part, *s* is the number of spares held and *q* is the cost of a vehicle. This function includes the cost of having additional vehicles in the fleet to allow for the number of grounded vehicles due to Back Orders. If we can reduce the number of radio Back Orders then we can reduce the size of the vehicle fleet.

Using a spreadsheet we can compute values for *EBO(s)* using the recursive relationship shown earlier. If the cost of the radio is $1,000 and the cost of the vehicle is $20,000 then the optimum number of spares to minimise this cost function is 14, which produces an *EBO* = 0.084.

Conclusion

Reliability centred sparing is a mathematical strategy to provide the optimum number of spares to meet the key system performance requirements. The mathematics depends on the support model implemented for the system and also the failure profile of the system.

This SERT Sheet has considered single depot sparing and SERT Sheet 1-5 will examine how the model changes for a multi-echelon sparing model.

References

1. Svanberg, K., *Optimization and Systems Theory*, KTH Stockholm, Sweden.

2. *Sherbrooke, C., METRIC: A Multi-Echelon Technique for Recoverable Item Control, RAND Memorandum RM-5078-PR, Nov 1966.*

About the Author:

Dr Ian Brace is a systems engineer with a background in communication systems and signal processing. He has consulted widely to industry and government in the areas of systems engineering and capability analysis and he has over twenty years experience in the defence domain. [Ian.Brace@systemdesigngroup.com.au]