Attribute Sampling Plans

Introduction

Attribute sampling involves constructing a plan to test whether a batch of items meet a certain pass or fail criterion. For example, in a batch of light bulbs does the sample bulb work or not? In a packaging process does the product weigh at least a kilogram? In a communication system are calls connected within less than five seconds? Since the decision is binary (pass/fail) this type of problem can be modelled using the Binomial probability distribution. This SERT Sheet will explain how develop attribute sampling plans that meet both the producer's and consumer's risk profile.

Attribute Sampling

When developing an acceptance sampling plan we typically use the binomial distribution. This distribution applies to events that can be given a pass or fail criterion, which is typically some form of attribute test.

Let's assume that we are going to test *n* items and the true probability of attribute failure is *p*. Then the probability that exactly *k* failures will occur in the batch of *n* items is given by:

$$\Pr(X = k) = \binom{n}{k} p^k (1-p)^{n-k} \text{ where } \binom{n}{k} = \frac{n!}{k!(n-k)!}.$$

This is simply the probability of *k* items failing and *(n-k)* items not failing, when all possible groupings of *k* items are included. However, if we characterise *passing the test* as finding not more than *k* failures in a batch of *n* items, then we need to calculate the probability of either zero failures, or a single failure, or two failures, or three failures, up to *k* failures, since any of these outcomes would constitute a pass result. The probability of passing the test is therefore expressed as

$$\Pr(X \leq k) = \sum_{i=0}^{i=k} \binom{n}{i} p^i (1-p)^{n-i}$$

and is known as the Cumulative Binomial Distribution.

Designing a sampling test is essentially the same mathematics, except that we do not select *n* and *k* in advance. Instead, we set the desired probability of success for the trial and then we work backwards to select an appropriate *n* and *k* in order to meet that level of success. The value of *p,* the probability of a defective item, is normally specified in the requirement.

Example: Call Setup Times

Consider a new telephone network with a reliability requirement that *95% of calls must be connected within five seconds*. What this means is that for an individual call the probability of failure, which is a call that takes more than five seconds to connect, must be less than 5%, or *p* = 0.05. How many test calls are required in order to be sure that the system can meet this requirement? In making those test calls, how many failures can be tolerated before the 95% confidence is sacrificed?

Acceptable Quality Limit (AQL)

To make this decision we need to choose *how sure* we want to be that if the true failure rate is 5% then we will pass the test. We cannot be 100% sure, since that would require an infinite number of test calls, but we might decide to be 95% sure. Hence, we need to find values for *n* and *k* such that:

$$0.95 = \sum_{i=0}^{i=k} \binom{n}{i} 0.05^i (0.95)^{n-i}$$

The selected desired maximum failure rate, *p*=5%, is called the Acceptable Quality Limit (AQL). There are, in fact, an infinite number of solutions to this equation. For example, *n*=1 and *k*=0 or *n*=17 and *k*=2 will both satisfy this equation, providing 95% confidence that we will pass the test if the true underlying failure rate is 5% or less. For each combination of *n* and *k* an Operating Characteristic (OC) Curve illustrates the probability of passing the test, given the true underlying probability of call failure. Figure 1 illustrates this for the example in which *n*=17 and *k*=2.

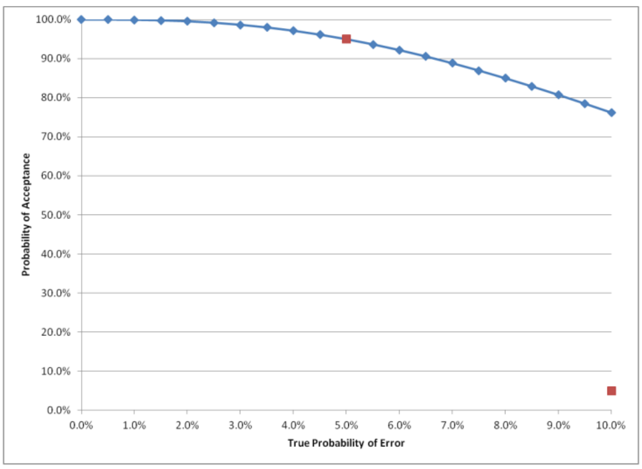

Figure 1 - OC Curve for n=17, k=2

The advantage of the OC Curve is that we can also see the problem associated with this test combination of *n*=17 and *k*=2. The curve shows that if the true probability of failure is 5% then the probability of passing the test is 95%, as desired. Unfortunately, if the true probability of failure is 10% then there is still a 76% probability of passing this test, which is probably an unacceptable risk for the customer.

Lot Tolerance Percent Defective (LTPD)

The customer might demand that if the underlying probability of call failure is 10% then the probability of passing the test should be no greater than 5%. In other words, a poorly performing system is likely to fail the test. This lower limit, *p*=10%, is called the Lot Tolerance Percent Defective (LTPD) and represents the lower tolerance limit of acceptable test results. To satisfy the AQL and also the LTPD we now need to find values for *n* and *k* that simultaneously satisfy both equations:

$$0.95 = \sum_{i=0}^{i=k} \binom{n}{i} 0.05^i (0.95)^{n-i}$$

$$0.05 = \sum_{i=0}^{i=k} \binom{n}{i} 0.1^i (0.9)^{n-i}$$

Using a spreadsheet we conclude that the test requires *n*=285 and *k*=20, as shown in Figure 2.

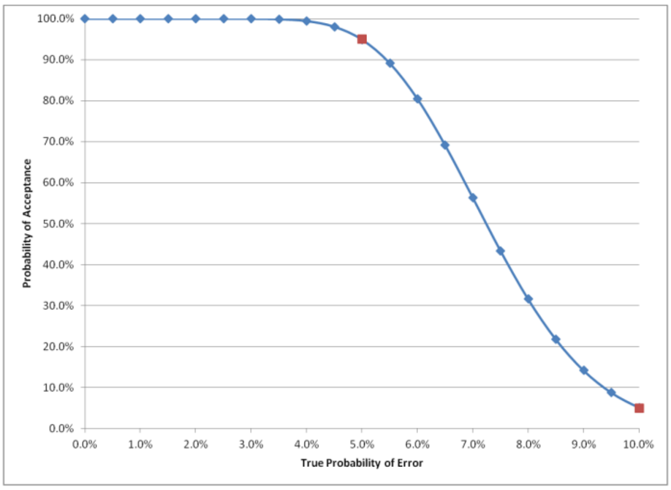

Figure 2 – OC Curve n=285, k=20

Conclusion

Attribute sampling plans use the Binomial distribution to determine the test parameters. By applying the distribution to both the AQL and LTPD simultaneously we can develop a sampling plan that will deliver the required level of confidence for both the producer and the consumer.

Note that enforcing both requirements requires a much more powerful test than only enforing one requirement.

This sort of planning should be performed before reliability requirements are released in a specification, since they indicate the magnitude of the associated test program required to achieve them. Generally, the closer together the AQL and LTPD, the larger the sampling plan must be.

About the Author:

Dr Ian Brace is a systems engineer with a background in communication systems and signal processing. He has consulted widely to industry and government in the areas of systems engineering and capability analysis and he has over twenty years experience in the defence domain.